Surface Impedance for Scattered Field on a Dielectric-Coated Circular Cylinder

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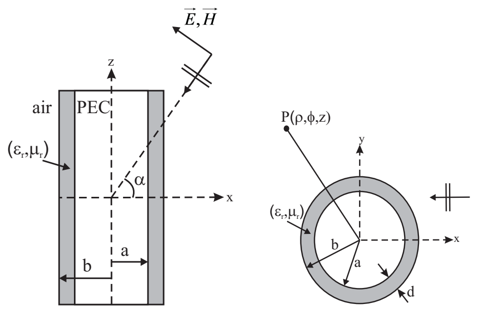
In the scattering analysis of a circular cylindrical structure, the impedance boundary condition (IBC) can approximate and simplify the perfect electric conductor (PEC) boundary condition. The circular cylinder problem can be solved with modal methods but they require a large number of terms when the cylinder radius is large in terms of the wave length. The uniform theory of diffraction (UTD) [1] is commonly used to overcome this issue. The two-dimensional problem of scattering on a circular cylinder covered by a dielectric layer has been analyzed by [2]–[5], but their solutions either do not consider oblique incidence, fail on the transition region or use a constant surface impedance.

This paper introduces a UTD solution for scattering on an impedance circular cylinder illuminated by an obliquely incident plane wave, where the surface impedance characterization turns out to be very important if a dielectric coating is considered. Such a surface impedance must depend on the cylinder geometry, the dielectric parameters and the wave numbers. The geometry of the problem is depicted on Fig. 1, a metallic circular cylinder of radius a covered by a dielectric layer of thickness d, where a plane wave is incident with an angle α. The observation point P, where the scattered field is calculated, is assumed to be far away from the cylinder surface.

The asymptotic expansion is performed over the vector potentials in spectral domain. The eigenfunction series solution is calculated by enforcing the boundary conditions on the cylinder surface, where the IBC in spectral domain is



(a) Side view. (b) Top view.

Fig. 1. Plane wave at oblique incidence on an infinitely long dielectric-coated PEC circular cylinder.

$$\frac{\partial \tilde{E}_z}{\partial \rho} - j\frac{k_{\rho 0}^2}{k_0}\Lambda_s^m \tilde{E}_z = j\frac{nk_z}{k_0 Y_0 \rho}\tilde{H}_z|_{\rho=b} \tag{1a}$$

$$\frac{\partial \tilde{H}_z}{\partial \rho} - j\frac{k_{\rho 0}^2}{k_0}\Lambda_s^e \tilde{H}_z = -j\frac{nk_z}{k_0 Z_0 \rho}\tilde{E}_z|_{\rho=b} \tag{1b}$$

where $\Lambda_s^{m,e}$ is defined as the normalized TM/TE surface admittance/impedance $\Lambda_s^m = \frac{Y_s^m}{Y_0}$ and $\Lambda_s^e = \frac{Z_s^e}{Z_0}$, with $Z_0 = \frac{1}{Y_0}$ the characteristic impedance of free space. If a constant surface impedance Z_s is considered then $\Lambda_s^e = (\Lambda_s^m)^{-1} \equiv \Lambda_s = \frac{Z_s}{Z_0}$. When $\Lambda_s \to 0$ the non-coated case is recovered.

Vector potentials are expressed in terms of Bessel and Hankel functions, and unknowns derived by solving the equation system stated by the boundary conditions in (1). The scattered field whether for the impedance or the coated cylinder can be expressed in terms of

$$q_{m,e}(n,k_z) = -j\Lambda_s^{m,e}(n,k_z)\cos\alpha \tag{2a}$$

$$q_c^{ibc}(n,k_z) = -j\frac{n}{k_0 b}\tan\alpha \tag{2b}$$

$$q_c^{coated}(n,k_z) = -j\frac{n}{k_0 b}\tan\alpha\left(\frac{\mu_r\varepsilon_r - 1}{\mu_r\varepsilon_r - \sin^2\alpha}\right) \tag{2c}$$

where superscript ibc refers to a constant surface impedance and $coated$ to a dielectric coating case.

Differences on the scattered field for the impedance and the metallic coated circular cylinder are in these $q_{m,e}$ and q_c parameters. For the impedance cylinder q_m and q_e represent the surface impedance dependency for a TM and a TE incident field at normal incidence, respectively, where $q_c = 0$. At oblique incidence, both modes contribute into the scattered field and are coupled through the q_c term. The effective surface impedance must change with the angle of incidence α and depend on the spectral variables n and k_z. By making a comparison between both problems the normalized surface admittance/impedance can be written in terms of products and ratios of Bessel functions as

$$\Lambda_s^m(n,k_z) = -j\frac{\varepsilon_r k_0}{k_{\rho 1}}\frac{J_n(k_{\rho 1}a)Y_n'(k_{\rho 1}b) - J_n'(k_{\rho 1}b)Y_n(k_{\rho 1}a)}{J_n(k_{\rho 1}a)Y_n(k_{\rho 1}b) - J_n(k_{\rho 1}b)Y_n(k_{\rho 1}a)} \tag{3a}$$

$$\Lambda_s^e(n,k_z) = -j\frac{\mu_r k_0}{k_{\rho 1}}\frac{J_n'(k_{\rho 1}a)Y_n'(k_{\rho 1}b) - J_n'(k_{\rho 1}b)Y_n'(k_{\rho 1}a)}{J_n'(k_{\rho 1}a)Y_n(k_{\rho 1}b) - J_n(k_{\rho 1}b)Y_n'(k_{\rho 1}a)} \tag{3b}$$

These equations are enough to be used in the eigenfunction solution, but for its implementation in the asymptotic approximations, Bessel functions of complex order are needed. Thus, to simplify equations (3), two-term Debye's asymptotic formulas are applied and a Taylor Series expansion around $1/b = 0$ is performed, where only the first two terms are retained [6]. This approach is valid for a thin dielectric coating, at least, for $d < 0.25\lambda_0$. Above this threshold, Olver's uniform formulation can be applied.

Finally, the UTD scattered field in the shadow region for the n-th creeping wave field contribution can be written in a compact way as

$$\begin{bmatrix} E_\theta^\pm \\ E_\phi^\pm \end{bmatrix} = \begin{bmatrix} D_m^\pm & D_c^\pm \\ D_c^\pm & D_e^\pm \end{bmatrix} \begin{bmatrix} E_\theta^i(Q_{a,b}) \\ E_\phi^i(Q_{a,b}) \end{bmatrix} \frac{e^{-jk_0 s}}{\sqrt{s}} \tag{4}$$

where $D_{m,e}$ and D_c are the TM/TE and coupled diffraction coefficients, respectively, $E_{\theta,\phi}^i(Q_{a,b})$ is the incident field on the cylinder surface and s is the ray length. And in the lit region is given by

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} E_\theta^i(P_L) \\ E_\phi^i(P_L) \end{bmatrix} + \\ + \begin{bmatrix} R_m & R_c \\ R_c & R_e \end{bmatrix} \begin{bmatrix} E_\theta^i(Q_R) \\ E_\phi^i(Q_R) \end{bmatrix} \sqrt{\frac{\tilde{\rho}^\gamma}{\tilde{\rho}^\gamma + l\cos\alpha}} e^{-jk_0 l} \tag{5}$$

with $R_{m,e}$ and R_c the TM/TE and coupled reflection coefficients, respectively, $E_{\theta,\phi}^i(P_L)$ is the incident field over the observation point, $E_{\theta,\phi}^i(Q_R)$ is the incident field on the surface reflected point, l is the ray distance and $\tilde{\rho}^\gamma$ is the reflected ray caustic distance to the point of reflection.

Fig. **??** and **??** show the UTD with IBC solution for the scattered field on a dielectric-coated PEC circular cylinder compared with the corresponding eigenfunction series solution. As expected, the UTD solution keeps uniform along the transition region, where GTD fails. When Elliot mode is present, because is a slowly attenuated wave and due to the cylinder curvature, it contributes strongly to the field for a TE incident plane wave and multiple encirclements need to be included.

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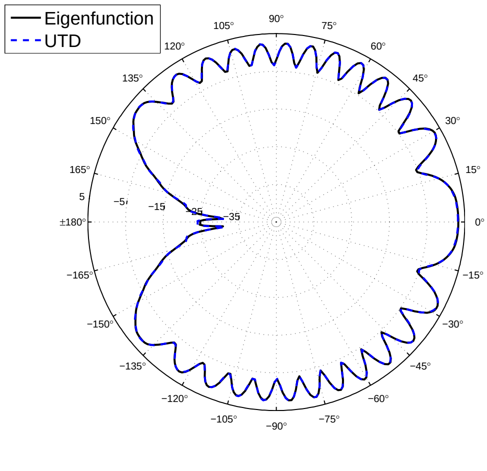
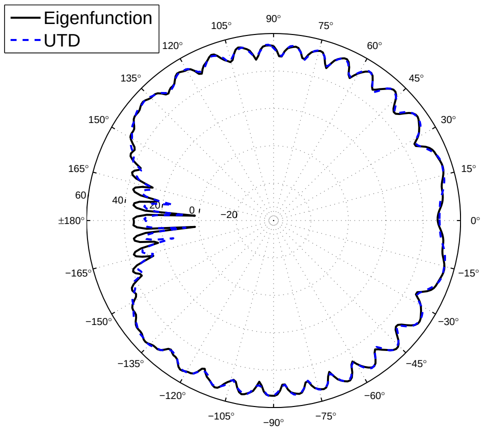


(a) TM incident field.



(b) TE incident field.

Fig. 2. Scattered electric field as a function of ϕ on a dielectric-coated grounded circular cylinder, with $\varepsilon_r = 3$, $\mu_r = 1$, $\alpha = 40^\circ$, $k_0 b = 30$, $k_0\rho = 80$ and $d = 0.1\lambda_0$.

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